Identification of Nano-Beams Rigidity Coefficient: A Numerical Analysis Using the Landweber Method

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Abstract: Due to their supporting function, beams are one of the main elements in structural projects. With the intense technological development in the field of nanotechnology, beams at micro- and nanoscales have become objects of intense study and research interest, see for example [8]. In this approach, we analyze numerically the inverse problem of identifying the stiffness coefficient in micro-nano-beams as a function that implicitly depends on the fractal media map for the continuum from strain measurements. Such a problem is unstable with respect to noise in strain measurements, which is inherent in practical problems. We introduce the equations that compose Landweber's iterative regularization method as a strategy to obtain a stable and convergent approximate solution with respect to the noise level in the measurements. We show some scenarios with simulated data for identifying the stiffness coefficient for different noise levels in measurements and for different coefficient of transformation of fractal medium. The results found numerically show that Landweber's method is a regularization strategy for the problem of identifying the stiffness coefficient in micro/nano-beams.

Keywords: micro, nano-beams, inverse problem, fractal media, Landweber's method.

I. Introduction

Every day, we are surrounded by beams. They are the fundamental structural elements that carry vertical loads. Though beams are traditionally used to describe building or civil engineering, beams can be found in all existing structures as structural elements, including machine frames, bones, carbon nanotubes, molecular chains, and other mechanical or structural systems. In these structures, the size-scale is paramount for a precise description of the mechanical properties of the beam [9].

In continuum mechanics, the analysis of movements and deformations is determined by the hypothesis that the medium is composed of matter in a homogeneous way. This theory ignores the existence of voids formed when molecules and atoms are not evenly distributed. The question that arises in micro- and nanoscale analysis is whether conventional models of continuum mechanics, such as Euler-Bernoulli and Timoshenko, may not be appropriate, given that such approaches do not take into account the scale factor in their models, see for example [9]. To address the scale issue, some non-classical continuum theories, as well as theories incorporating non-integer order derivatives in the [6, 7, 8, 9] models, have been investigated. Models with fractional dynamics have been shown to be more suitable for describing the properties of various real materials, e.g. [6], and thus have aroused the interest of engineering research.

In [7] an overview of the modeling of fractal media through the theory of continuous mechanics is presented using the ideas proposed in [8]. This theory consists of describing the laws of equilibrium for fractal media using fractional integrals. Using a map from the fractal to the continuous medium, those fractional integrals are rewritten as integrals in conventional Euclidean space. The interesting thing about this approach is that the essential condition of continuum mechanics, the separation of scales, can be replaced by the use of continuum field equations. In Section II, we present the deduction of the Euler-Bernoulli equation for beams in fractal media, using the techniques proposed in [8]. We also show that the analyzed model has a unique solution $u(x)$, which is known in the literature as the direct problem for the Euler-Bernoulli beam model in fractal media.

The main contribution of this work is the numerical investigation of an "inverse problem" for the Euler-Bernoulli beam model in fractal media. Indeed, the stable identification of the stiffness parameter $a(x)$ associated with the Euler-Bernoulli equation in fractal media, as described in Section II, from indirect measurements of the nano (micro)-beam deflection of $u(x)$.

Given that measurements of the nano (micro)-beam deflection $u(x)$ are subject to errors and that the inverse problems are generally ill-posed in the Hadamard sense [2, 3], the issue of instability in the identification of the stiffness parameter $a(x)$ due to noise measurements of the beam deflection $u(x)$ necessitates the use of some regularization strategy. In this contribution, we use the Landweber iterative method (see Section III), which will be used to numerically demonstrate the stability of the approximations for the identification coefficient $a(x)$ in Section IV. In Section IV, we will present several numerical tests with varying levels of noise in the measurements. The presented scenarios demonstrated numerically that the Landweber iteration obtains stable approximate solutions for the coefficient $a(x)$ under different fractal medium properties.

II. Euler-Bernoulli Equation in Fractal Media

In general, a fractal medium cannot be considered as a continuous medium, as there are points and domains that are not filled by particles of the medium. These domains can be called porous. Thus, the application of continuum theory to fractal media is not appropriate. To get around this difficulty, [8] proposed the use of fractional integrals to represent the mass of a region Importar tabla Importar tabla in three-dimensional Euclidean space E^3 tabla as being:

$$m(W) = \int_W \rho(R)dV_D = \int_W \rho(R)c_3(D,R)dV_3 \tag{1}$$

with

$$c_3(D,R) = R^{D-3}\frac{2^{3-D}\Gamma(3/2)}{\Gamma(D/2)}, \quad R = \sqrt{x_i x_i}$$

(2)

where R is the length scale measurement, D is the fractal dimension of mass, and is Euler's Gamma function. The coefficient c_3 establishes the transformation between fractal and continuous media. The term dV_D tabla is the infinitesimal element of volume in fractal space, and dV_3 is the infinitesimal element of volume in E^3. However, the proposal of [8] implies that the measure of the fractal dimension in each of the directions of the medium must be the same. To contour this limitation, [8] proposed an approach in which the measure of length in relation to each coordinate of the medium is given by:

$$dl_k(x_k) = \rho(x)c_1^{(k)}(\alpha_k, x_k)dx_k, \quad k = 1,2,3, \tag{3}$$

where $\rho(x)$ represents mass density and a fractal dimension in the direction x_k

Assuming that $c_1^{(k)}$ is given by the modified Riemann-Liouville integral, that is:

$$c_1^{(k)} = \alpha_k\left(\frac{l_k - x_k}{l_{k0}}\right), \qquad k = 1,2,3. \tag{4}$$

where l_k is the total length along x_k and l_{k0} is the characteristic length in the given direction, [8] showed that it is possible to reproduce almost all the known results of the mechanical theory of the continuum, in addition to allowing to represent more heterogeneous media.

A. One-dimensional fractal medium: the Euler-Bernoulli equation

Assume that we are in Euclidean dimension 1, in the direction. Let dimension $D < 1$ be the dimension of the fractal structure in which we are immersed, in the direction x. Then it follows from (3) that the element must be replaced by:

$$dl_D = c_1(D,x)dx. \tag{5}$$

Rewriting the balance equations in variational form, with the measure given by (5) and using the generalized Green-Gauss Theorem, see [8], it follows that the Euler-Bernoulli equation in fractal means is given by:

$$\nabla_x^D\nabla_x^D(EI\nabla_x^D\nabla_x^D u) = 0 \tag{6}$$

wherein

$$M(x) = a(x)\nabla_x^D \nabla_x^D u(x) \tag{7}$$

is the bending moment with $a(x) = E(x)I(x)$ the stiffness coefficient. In (6) and (7), the operator $\nabla_x^D \nabla_x^D$ is the Laplacian operator for fractal media, given by:

$$\nabla_x^D \nabla_x^D u = c^{-1}(c^{-1}u'(x))' \tag{8}$$

where $c = c(x) = c_1(x)$ is the transformation coefficient between the fractal medium and the continuous medium.

In this work, we will consider (7) with the following boundary conditions:

$$u(0) = u'(0) \tag{9}$$

corresponding to a cantilever beam.

B. The inverse problem as an equation of operators

To formulate the inverse problem that we are interested in this work, we first need to formulate some hypotheses for which problem (7) with boundary conditions (9), has a single solution, that is, the direct problem is well posed.

A1: The stiffness coefficient $a = a(x)$ and the transformation coefficient $c = c(x)$ are measurable functions on that satisfy the condition $\bar{a} \geq a(x) \geq \underline{a} > 0$ and for known constants $\bar{a}, \underline{a}, \bar{c}, \underline{c}$. The set of coefficients satisfying the hypothesis **A1** will be denoted by *Ad* in this manuscript and referred to as the admissible set

A1': The stiffness coefficient and the transformation coefficient besides satisfying **A1** have uniformly bounded $||a'||, ||c'||$, that is, the coefficients belong to the set $A = \{a, c \in L^\infty | \lambda_1 \leq a, c \leq \lambda_2, ||a', c'|| \leq Q\}$, where $||.||$ denotes the norm $L^2(0, L)$ with $0 < \lambda_1 \leq \lambda_2 < \infty$ and $Q > 0$.

A2: The bending moment $M(x) \in C[0, L]$.

Consider the vector space

$$\mathcal{L}^2[0, L] := \{f$$

measurable, such that

$$\int_0^L c^{-1}(x)(f(x))^2 dx < \infty\}$$

(10)

with the inner product below

$$< f, g >_{\mathcal{L}^2} = \int_0^L \sqrt{c(x)}f(x)\sqrt{c(x)}g(x)dx$$

(11)

It follows from the Assumption **A1** that the space of functions $L^2(0,L)$ with the norm induced by the inner product (11) is a space of Hilbert. Furthermore, the space $\mathcal{H}_0^1[0,L]$ will denotate the Sobolev space of all functions in $g \in \mathcal{L}^2[0,L]$, with the derivatives in the weak sence, g' also belongs to $\mathcal{L}^2[0,L]$, and satisfies $g(0) = g(L) = g'(0) = g'(L) = 0$. See, for example [1].

In order to prove existence and uniqueness for a solution of the problem (7) with conditions in (9), we use the theory of weak solution as follows. First, we consider as a weak solution to problem (7) any function $u \in \mathcal{H}_0^1[0,L]$ such that

$$\int_0^L c^{-1}u'\phi' dx = \int_0^L \frac{M}{a} c\phi dx$$

(12)

for any test function

Lemma 1: Assume that the assumptions **A1** and **A2** are satisfied. If there is satisfying the problem (7) and (9), then is a weak solution to the problem (12). Conversely, if is a weak solution to (12), then satisfies (7) almost always.

Proof: It follows from Hypothesis **A1** and **A2** that $cM/a \in L^2[0, L]$. Therefore, from (7) and conditions (9), $u \in \mathcal{H}_0^2[0,L]$ and satisfy (12) (see the Green's identities in [1]).

Reciprocally, if $u \in \mathcal{H}_0^1[0,L]$ is a weak solution to (7), it is, satisfies (12), then boundary conditions (9) for the problem in (7) are satisfied. Furthermore, as u satisfied (12) by assumption, we have, after an integration by parts, that

$$< -a\nabla_x^D \nabla_x^D u - M, \phi > = < 0, \phi > \forall \phi \in C_0^\infty[0, L]$$ (13)

Therefore, it follows from the density of $C_0^\infty[0,L]$ in $\mathcal{H}_0^1[0,L]$ that $-a\nabla_x^D\nabla_x^D u = M$, almost everywhere. Hence, by the Hahn-Banach theorem (see [1]) the result for (13) can be extended to $\mathcal{H}_0^1[0,L]$.

Given the Lemma 1, it is possible to prove through the Lax Milgram's Theorem [1] the existence of a unique solution $u \in \mathcal{H}_0^1[0,L]$ that satisfies (12). Indeed, notice that is a weak solution of (7) if and only if u satisfies

$$A(u, \phi) = l(\phi) \qquad \forall \phi \in \mathcal{H}_0^1[0, L]$$

(14)

where $A : H_0^I[0, L] \times H_0^I[0, L] \to \mathrm{R}$ is the bilinear form defined as

$$A(u, \phi) := \int_0^L c^{-1} u' \phi' dx \tag{15}$$

and $l : H_0^I[0, L] \to \mathrm{R}$ is the linear functional given by

$$l(\phi) := \int_0^L \frac{M}{a} c\phi dx. \tag{16}$$

Theorem 1: Assuming that the hypotheses **A1** and **A2** are satisfied, there is a unique solution to (12). As a consequence of the Lemma 1, there is a unique weak solution to (7).

Sketch of Proof: Following the same ideas in [4], it is possible to prove that the linear functional defined in (16) is continuous and furthermore that the bilinear form defined in (15) is continuous and coercive in $\mathcal{H}_0^1[0, L]$. Therefore, it follows from the Lax-Milgram Theorem [1, Corollary 5.8] the existence of a unique function $u \in \mathcal{H}_0^1[0, L]$ satisfying (12).

As a result of Theorem 1, it follows that, for any given function $c(x)$, satisfying Assumption **A1**, the operato

$$\begin{aligned} F_{c(x)} : Ad \subset \mathcal{L}^2[0, L] &\longrightarrow \mathcal{L}^2[0, L] \\ a &\longmapsto u(a) \end{aligned} \tag{17}$$

where $u(a) = u_c(x, a(x))$ is the unique solution of (7), is well defined.

$F_{c(x)}$ is called the forward operator in the theory of inverse problems, see for example [2, 3].

C. The inverse problem

Assume that the functions $M(x)$ and $c(x)$ are known. The inverse problem that we are interested in this work deals with the identification of the stiffness coefficient $a(x)$ from measurements $u^\delta(x)$, with noise level δ, satisfying:

$$||u^\delta - u_c(a)||_{\mathcal{L}^2[0,L]} \leq \delta \tag{18}$$

of the deflection $u(x) = u_c(x)$ solution of (7) with boundary conditions (9). Equivalently, determine $a(x)$ in the operator equation (17), from the measures $u^\delta(x)$ satisfying (18).

Inverse problems, in general, do not have the property of continuous dependence of the measures u^δ. This implies that small perturbations of magnitude δ in the measurements can generate large perturbations in obtaining the solution of the inverse problem of interest, e.g., [2, 3]. As a result, obtaining stable and convergent solutions with respect to the noise level δ requires the use of regularization methods. See for example [2, 3, 4, 5].

The problem of identifying the stiffness coefficient $a(x)$ in a beam does not have the property of continuous dependence of the measures u^δ, as demonstrated in the case of $c(x) = 1$ in [4, 5]. As a result, the stable identification of the stiffness coefficient $a(x)$, requires some regularization methods [2, 3]. In this contribution, we will use an iterative regularization method called the Landweber method [2] to recover the parameter $a(x)$ in a stable and convergent manner with respect to the noise level δ. In other words, we will show numerically that the Landweber iteration (see equation (19)) together with a stop criterion, called the discrepancy principle (see equation (20)), generates approximate stiffness coefficients a_k^δ, for $a(x)$, with values that are stable and convergent to $a(x)$, as a function of the noise level in the data δ. The iterative algorithm is presented in Section III, while the numerically simulated scenarios for the recovery of the stiffness coefficient $a(x)$ is presented in Section IV.

III. Landweber's Iterative Method

The Landweber iteration (Landweber's iterative method) for the identification of the coefficient in (7), is given by

$$a_{k+1}^\delta = a_k^\delta + \gamma F'_{c(x)}(a_k^\delta)^*(u^\delta - F_{c(x)}(a_k^\delta)) \tag{19}$$

where γ is a relaxation parameter. $F'_{c(x)}(a_k^\delta)^*$ denotes the adjunct of the Fréchet derivative of the parameter-to-measurement operator $F_{c(x)}(a)$, defined in (7). is the initial guess of the iteration (14), that shall be chosen properly.

Because the data contains δ noise, the iterative method must be combined with a stop rule, as mentioned in [2, 3, 4, 5]. In this work, we use the discrepancy principle's stopping criterion, which states that (19) must be stopped at the first step k^* that satisfies

$$||u^\delta - F_{c(x)}(a_{k^*}^\delta)|| \le \tau\delta < ||u^\delta - F_{c(x)}(a_k^\delta)|| \tag{20}$$

for some $\tau > 1$. Thus, the number of iterations determines the stopping rule of the method.

The numerical implementation to obtain the coefficient $a(x)$ iteratively according to (19) is given by the following algorithm:

(1) Choose an initial value for $a_0 \in L^\infty$ and $c(x)$ satisfying Assumptions **A1** and **A1'**, respectively. Choose also the parameter values δ, τ, γ

(2)Add the uniformly distributed random variable $z(x)$ to [0,1] the solution $u(x)$ of the forward problem to generate the noise data $u^\delta = u(x) + \delta z(x)$, satisfying $|u(x) - u^\delta| \le \delta$

3) As long as the iteration (19) is such that a_k^δ , for$k < k^*$, where k^* denotes the iteration index that satisfies the discrepancy principle, do the following steps:

(4) Solve the problem with the initial conditions

$$a_k^\delta c^{-1}(c^{-1}u'(a_k^\delta))' = M$$

with the initial conditions

$$u(0) = u'(0) = 0.$$

(5) Evaluate the residue

$$r_k = u_k^\delta - F_{c(x)}(a_k^\delta)$$

where in $F_{c(x)}\left(a_k^\delta\right)$ is solution of differential equation calculated in Step (4).

(6) To calculate $F'_{c(x)}\left(a_k^\delta\right)^* r_k$, firstly,

(6.1) Solve the differential equation

$$-(c^{-1}(a_k^\delta v_k)')' = r_k$$

with finals conditions

$$v(L) = v'(L) = 0.$$

(6.2) Then, find the adjunct

$$AD_k := F'_{c(x)}(a_k^\delta)^* r_k$$

solving

$$\frac{M}{a_k^\delta} v_k$$

wherein v_k is solution of equation obtained in Step (6.1).

(7) Update a_{k+1}^δ wherein $a_{k+1}^\delta = a_k^\delta - \gamma AD_k$

(8) Go back to Step (3) while the discrepancy principle given by (20) is not reached.

(9) Otherwise, the regularized solution is $a_{k^*}^\delta$, where k^* is deteremined by the discrepancy principle (20).

It is important to mention that, for the calculations of $F'_{c(x)}(a_k^\delta)^*$ in Step (6) in the algorithm, it is necessary to define an auxiliary operator given by

$$A(a) : H^2[0,L] \cap H_0^1[0,L] \longrightarrow L^2[0,L]$$
$$u \longmapsto A(a)u := -ac^{-1}(c^{-1}u')'. \tag{21}$$

for $a,c \in Ad$. It is straightforward to show that the operator defined in (21) is linear, bounded and bijective. Therefore, it also has a linear and bounded inverse $A^{-1}(a)$. The adjunct operator of $A(a)$ is such that $A^*(a)v = -(c^{-1}(av)')'$, corresponding the Step (6.1) in the algorithm. Because it is also linear and bounded, its inverse is given by

$$(A^*(a))^{-1}w = v \tag{22}$$

where is the unique solution of the

$$-(c^{-1}(av)')' = w.$$

The equations obtained in Step (6) of the above algorithm are calculated by taking the Fréchet derivative of the operator $F_{c(x)}(\cdot)$ defined in (12) and integrating by parts with respect to the inner product given by (11). Therefore, we obtain

$$F'(a)^*r = (c^{-1}u')'(A^{-1}(a))^*r \tag{23}$$

with $A^*(a))^{-1}$ as (22) applied to residue $r = u^\delta - F_{c(x)}(a^\delta)$

Hence, the Steps (6.1) and (6.2) of the algorithm are equivalent to (23).

IV. Numerical Examples

In this section, we use the Landweber regularization method given by (19) to identify the beam stiffness coefficient $a(x)$, in the fractal media Euler-Bernoulli equation modeled by (7) and (9). In all the simulations presented below, we use $x \in [0,1]$, and the bending moment $M(x) = \frac{x^2}{2} - x + \frac{1}{2}$. Also, we use $a_0 = 0.75$ as the initial guess of the Landweber iteration method (19). The finite difference method was used to obtain the numerical solution u for (7) at points $x_i = i/n$ where $i = 0,1,2,\ldots,n$ for $n = 50$ points for the simulated scenario of Example 1 and $n = 100$ points for the other simulated scenarios. In all examples, the noisy data u^δ is generated by adding a random variable $z(x) \in [0,1]$, evenly distributed, to the solution $u(x)$ of (7), such that $u^\delta(x) = u(x) + \delta.z(x)$, where δ is the noise level.

The differential equations corresponding to Step (6) of the algorithm were solved using backward Euler's method to account for the final conditions. The steps of the algorithm resulting from the Landweber method, presented in Section III, were implemented in Python (version 3.8.5).

Example 1: The simulated scenario corresponding to this first example consists in identifying the stiffness coefficient $a^*(x) = 1$ in the fractal medium Euler-Bernoulli beam where the fractal medium transformation coefficient is $c(x) = 2x + 1$.

Fig. 1 compares the coefficients $a(x)$ and $a^{\delta}_{k^*}(x)$ recovered by the Landweber method given by (14) for noise levels $\delta = 0.0001, \delta = 0.001$ and $\delta = 0.01$, respectively. The simulations were performed on a mesh with $n = 100$ points. The numerical results shown in Fig. 1 demonstrate that Landweber's iterative method produces stably approximate solutions $a^{\delta}_{k^*}(x)$ for the stiffness coefficient $a^*(x) = 1$, as a function of the noise level δ.

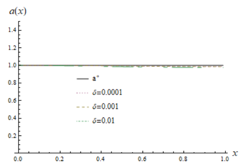

FIG. 1.
Identification of the stiffness coefficient *a*(x)* corresponding to Example 1

In the simulations presented in Fig. 1, the reconstructed coefficient $a^{\delta}_{k^*}(x)$ is obtained by using the discrepancy principle, for with the iteration is stopped after $k^* = 143$ iterations for the noise level $\delta = 0.0001$. While, the iteration is stopped after $k^* = 92$ and $k^* = 77$, for the noise level of $\delta = 0.001$ and $\delta = 0.01$, respectively.

In the following simulated scenarios, we will consider the identification of a non-constant stiffness coefficient , as a way of evaluating the performance of Landweber's iterative method in more unfavorable scenarios

Example 2: The simulated scenario of this example corresponds to the identification of the stiffness coefficient $a^*(x) = \frac{1}{2 - x}$, from noise measurements of the fractal Euler-Bernoulli beam equation (6), where the fractionality of the medium is given by $c(x) = 2x + 1$.

The results for noise levels of $\delta = 0.0001, \delta = 0.001$ and $\delta = 0.01$ are shown in Fig. 2. The mesh was chosen uniformly with $n = 50$ points. This figure show that the Landweber iteration with the stopping criterion given by the discrepancy principle in (20) produced stable and satisfactory approximations $a^{\delta}_{k^*}$ for the non-constant coefficient $a^*(x)$.

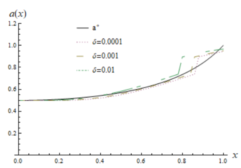

FIG. 2.
Identification of the stiffness coefficient for Example 2

The principle of discrepancy given in (20) is reached for the scenarios of this example with $k^* = 101$, $k^* = 44$ and $k^* = 37$, for the simulated noise levels for $\delta = 0.0001, \delta = 0.001$, and $\delta = 0.01$, respectively.

In the simulated scenarios that follow, we present approximate solutions for recovering the coefficient $a^*(x)$ as in Examples 1 and 2, where the fractionality of the medium $c(x)$ is distinct.

Example 3: The simulated scenario of this example corresponds to the Euler-Bernoulli fractional media beam, where the coefficient of fractionality is given by $c(x) = x^2 + 1$. The simulations for a noise level of $\delta = 0.0001, \delta = 0.001$ and $\delta = 0.01$, are presented in order to recover the coefficient $a^*(x) = 1$, in Fig. 3.

Fig. 3 shows the coefficient $a(x)$ recovered by the Landweber method given in (19) for different noise levels $\delta = 0.0001, \delta = 0.001$ and $\delta = 0.01$. The simulation was performed on a mesh with $n = 50$ points.

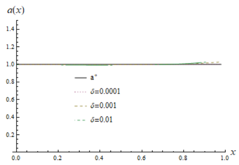

FIG. 3.
Identification of the constant stiffness coefficient for Example 3

In Fig. 3, the reconstructed coefficients $a^{\delta}_{k^*}(x)$ for different noise levels $\delta = 0.0001, \delta = 0.001$ and $\delta = 0.01$ satisfied the discrepancy principle, respectively, with $k^* = 102,\ k^* = 97$ and $k^* = 89$.

Example 4: The simulated scenario of this example corresponds to the Euler-Bernoulli fractional media beam, where the coefficient of fractionality is given by $c(x) = x^2 + 1$. The simulations for noise level $\delta = 0.0001, \delta = 0.001$ and $\delta = 0.01$ are presented in Fig. 4 in order to recover the coefficient $a^*(x) = \frac{1}{2-x}$

Fig. 4 shows the coefficient $a^{\delta}_{k^*}(x)$ recovered by the Landweber method given in (19) for different noise levels $\delta = 0.0001, \delta = 0.001$and $\delta = 0.01$ respectively. The simulation was performed on a mesh with $n = 50$ points.

The principle of discrepancy given in (20) is reached of this example for the simulated noise levels, where for for and for .

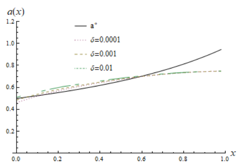

FIG. 4.
Identification of the stiffness coefficient for Example 4

The numerical results shown in Examples 1, 2, 3, and 4 demonstrate that Landweber's iterative method terminated with the discrepancy principle produces stably approximate solutions $a^{\delta}_{k}\cdot(x)$ for the simulated scenarios with the stiffness coefficient $a(x) = 1$ and $a^*(x) = \frac{1}{2-x}$ for different proposed noise levels with distinct fractionality transformation coefficient $c(x)$.

It is worth noting that the discrepancy principle in Examples 3 and 4 is stretched after more iterations than previous examples, which is possible due to the polynomial degree of the coefficient $c(x)$.

It will be investigated in future contributions.

V. Conclusions

In this paper, we present a fractal mechanics-based version of the Euler-Bernoulli equation for beams at micro- and nanoscales, as well as the inclusion of the parameter $c(x)$ responsible for characterization of the fractionality of scales. We investigated the inverse problem of identifying the Euler-Bernoulli equation coefficient $a(x)$ from measures of noisy data corresponding to the bending of a fractal media beam. As this problem is ill-posed in the Hadamard sense, we numerically analyze the Landweber iteration method as a regularization, in order to obtain stable and convergent solutions for the parameter of interest in terms of the noise level. We present some numerical examples for different noise levels in the recovery of constant and non-constant stiffness coefficients $a(x)$. In addition, we performed tests with two different functions for the fractal parameter $c(x)$, evaluating the performance of the method in these cases as well. In fact, the numerical results presented showed that the proposed iterative method satisfactorily recovered the stiffness coefficient, reaching the stopping criterion with a similar number of iterations in the different tests, even when simulated for different parameters $c(x)$.

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